Critical communication in primary mathematics

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What is mathematical reasoning?

"Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. Students are reasoning mathematically when they explain their thinking, when they deduce and justify strategies used and conclusions reached, when they adapt the known to the unknown, when they transfer learning from one context to another, when they prove that something is true or false, and when they compare and contrast related ideas and explain their choices"

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Teaching mathematical reasoning

Most teachers I talk to see mathematical reasoning as involving talking about how you got the answer.

- How do you assess the quality of students ‘talking about how they got the answer’?

- How do you teach young children to get better at ‘talking about how they got the answer’?

If you don’t have answers to these questions, then you’re probably going to get students to talk about how they got their answer and hope that this will lead to them developing mathematical reasoning.

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Overview for today:

- Practical approaches to assessing and developing young children's capacity to reason mathematically
- Think about the role that teachers can take in facilitating this development

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Always, sometimes, never

An easy format that builds students' reasoning capacity

1. Working in groups with coloured textas and a big sheet of paper

2. I'm going to put up a mathematical statement

3. As a group, please decide if the statement is always true, sometimes true, never true

Statement 1

When I multiply a number by 14, I always get a bigger number

Mathematical Reasoning

Capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising.

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Statement 2

A pentagon can have the same number of right angles as a rectangle

Proof by induction vs deduction

Is it easier to prove you're right by examples (induction) or can you find a rule that proves it right for all cases (deduction)?

Statement 3

The sum of three consecutive numbers is divisible by 3

Examples and counter examples

How much proof do you need if a statement is always true, sometimes true or never true?

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Benefits of this task…

- Teaches children how to prove a statement, and what level of proof is required
- Shows children that maths has a logic and the rules aren't random or just because the teacher says so
- Teaches students how to communicate with each other about mathematical ideas
- By working in groups, students who are less sure about the maths can hear others' arguments
- Each group produces written work that can support assessment of mathematical reasoning
- It helps teachers get better at mathematical reasoning

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Always sometimes never in a maths program

- As a way of dealing with misconceptions in a unit:
  The longer a decimal is, the bigger the number

- As stand alone activities

Used once or twice a week, these activities can have a significant impact on students' capacity to communicate critically in maths…

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A real life example

I'm going to use data from my Masters research to illustrate some points as we go along

I wanted to investigate mathematical reasoning and what teachers could do to develop it

I tested a cohort of Grade 5/6s and asked them to prove some mathematical statements:

e.g. "why do the angles inside a triangle equal 180°"?

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How did students respond?

‘No reason’ response: “I don’t know”

‘External conviction’ response: “because my teacher said so”

‘Inductive reasoning’ response: provides a few examples, then claims that this suggests that it’s probably true for all triangles

‘Deductive reasoning’ response: “if you double a triangle, it makes a parallelogram. Parallelograms have 360, so half a parallelogram is 180”

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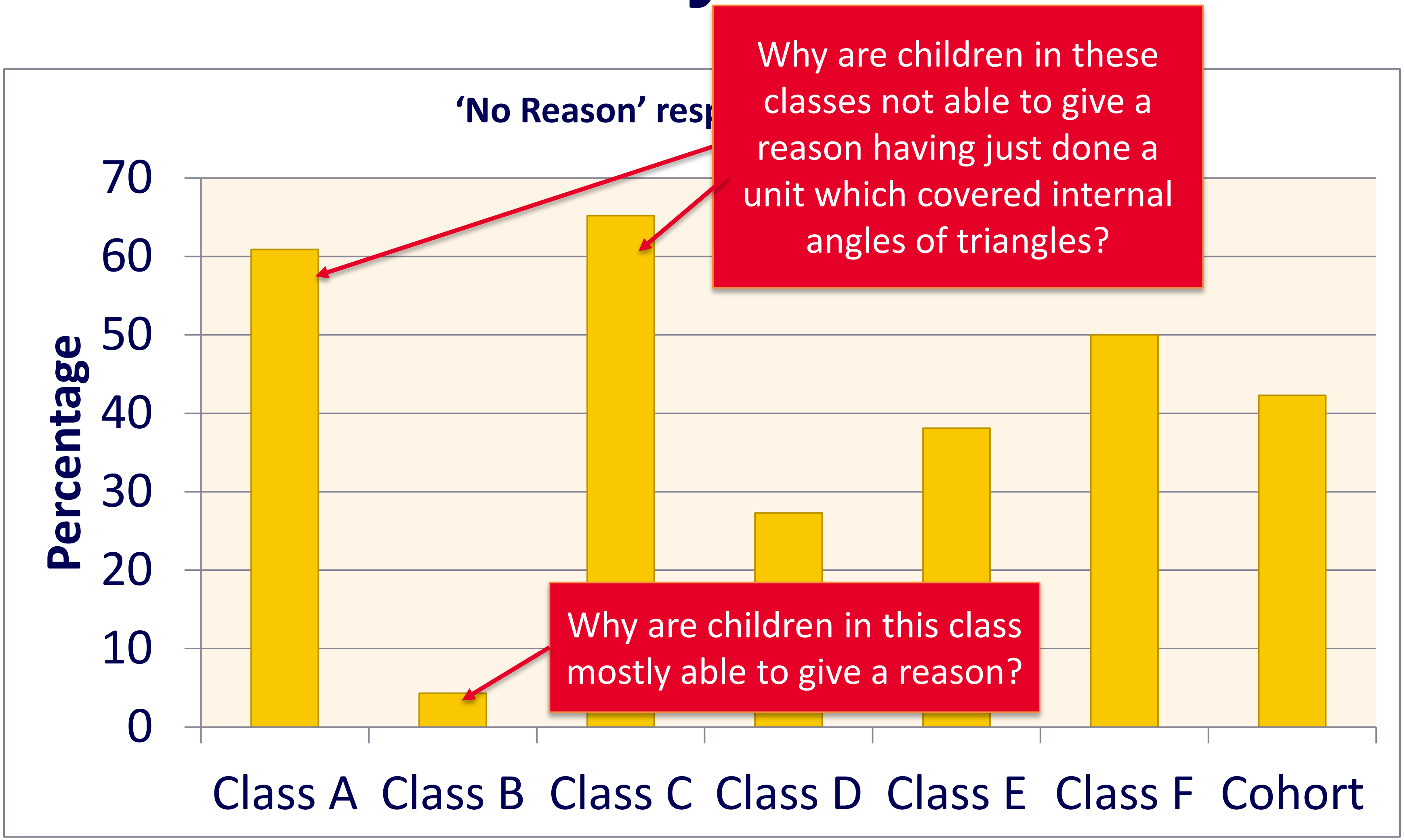
Results varied by class



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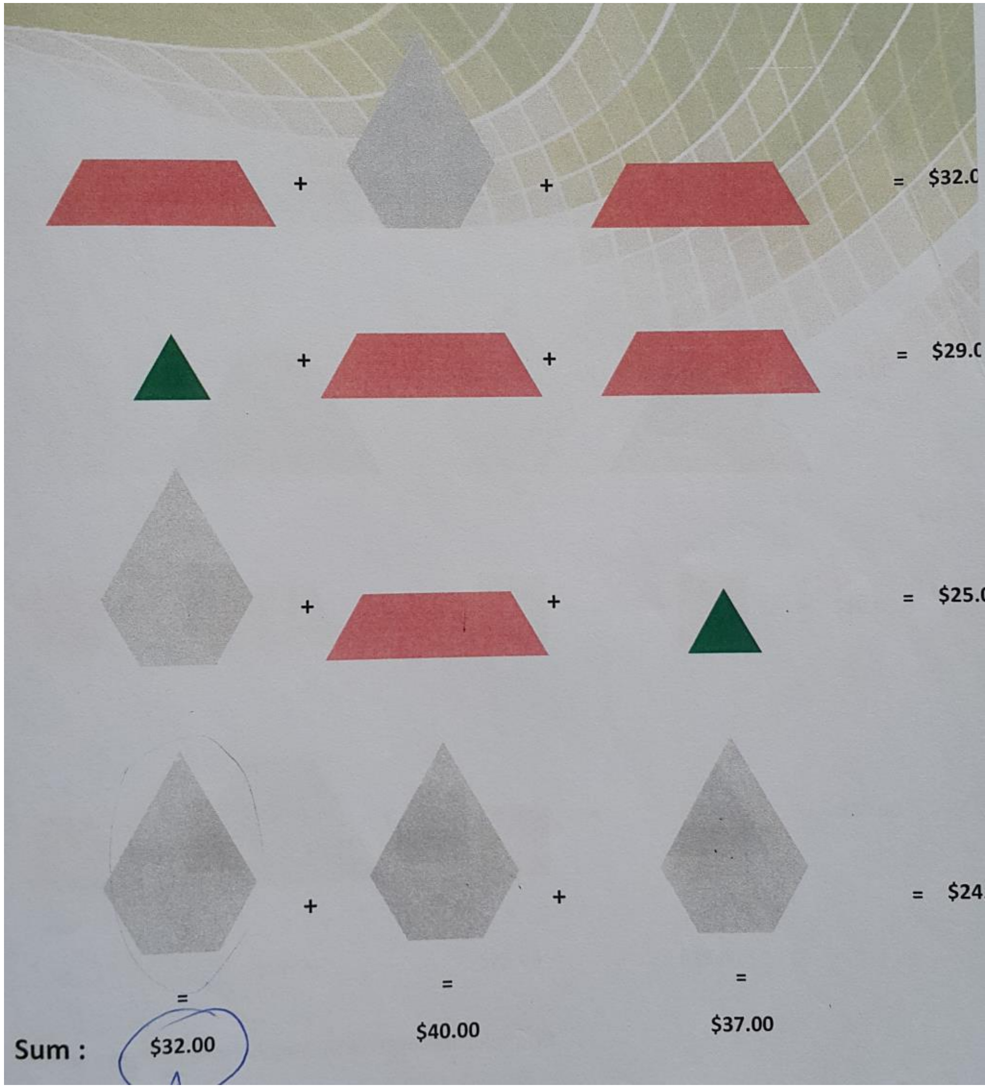
Videoing each teacher teaching the same lesson

As a team, the following task was chosen as the focus of a lesson which was filmed



What was different between Class A and Class B?

Over 60% of Class A could not give a reason for the triangle question

Only 5% of Class B couldn't give a reason for the triangle question

Is there something different about the way in which each class attempts the same lesson that could explain why students from Class B are better at mathematical reasoning?

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Lesson intro

<u>Mathematical Reasoning</u>
Students are reasoning mathematically when … they transfer learning from one context to another …

Both teachers shows childr
questions:

Class A
"how are we going to work this out?"

Class B
"Does this look like a problem we've done before?"
"So what did we have to do in those other problems?"
"Is there anything different about this problem?"

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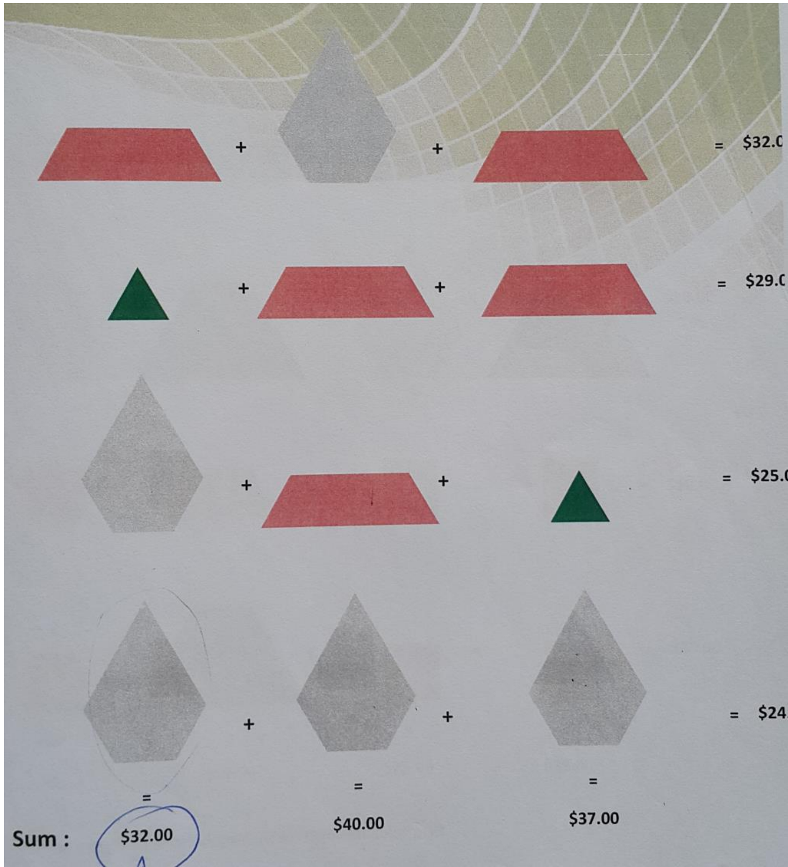
Students suggest solution strategies

In both classes, the teacher asked students to suggest how the problem could be approached

Class A – a student suggests that a pink block could be $5

Teacher response:
"No, no. You can't just start with a block. Pick a line".

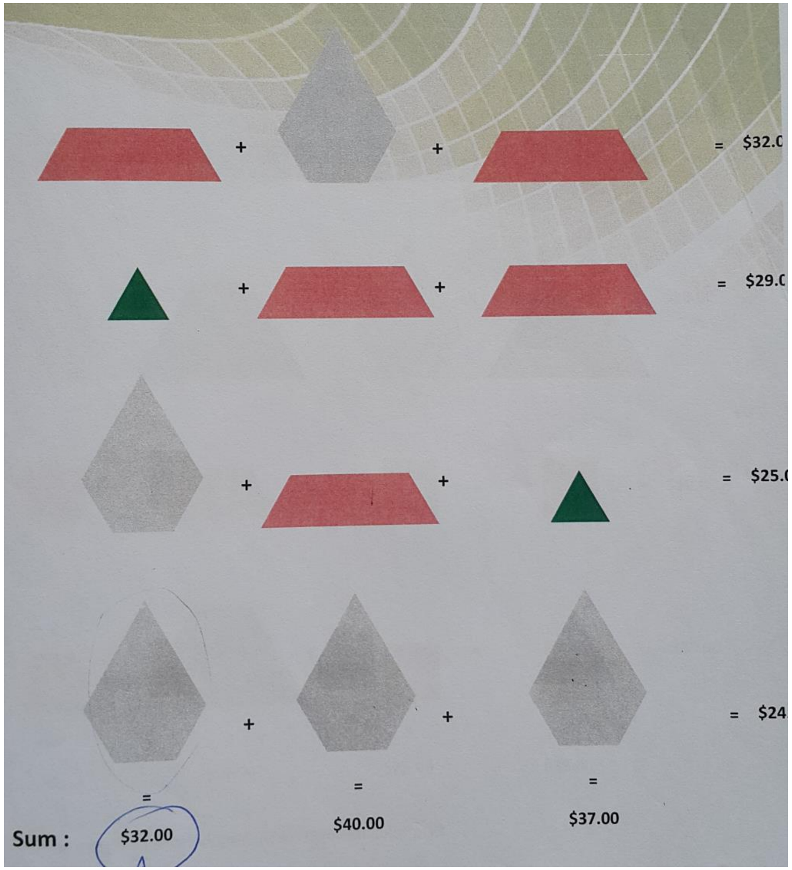


Students suggest solution strategies

Class B – a student suggests that a pink block could be $10

Teacher response:
"OK. Let's see what happens if the pink block was $10. If the pink block is $10, does that tell us what any of the other blocks is worth?"

After 2 minutes of class discussion, the 'picking a random number' strategy has been publicly evaluated



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Teacher as 'arbiter of correctness'

Harel and Rabin (2010) use the term 'arbiter of correctness' to describe a teacher role that limits students' capacity to learn mathematical proofs

- Students suggest ideas and the teacher determines whether the idea is correct or incorrect
- The mathematical validity of a student's idea is being evaluated privately in the teacher's head
- This means that students have less opportunity to evaluate, justify and explain their ideas

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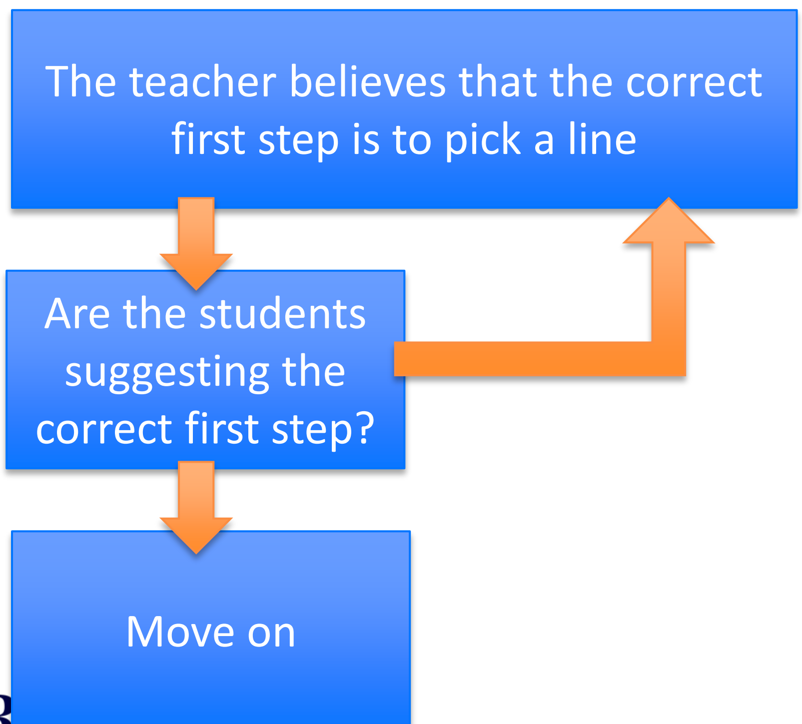
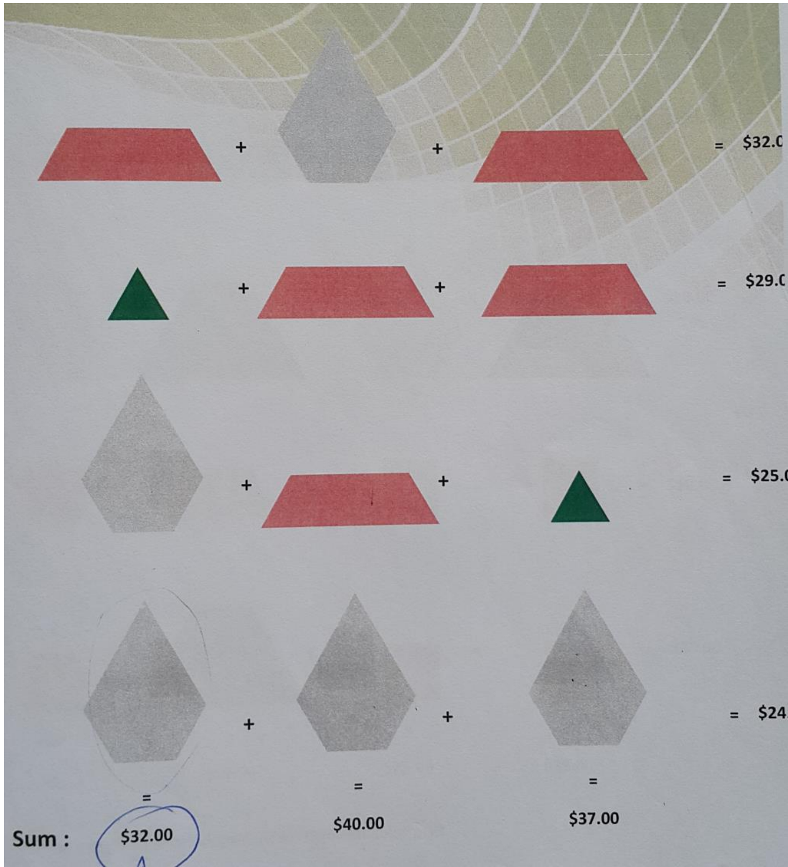
An example from Class A

Teacher: “how should we work it out?”

Student: “we could use division”

Teacher: “hmm. We can’t really divide anything yet, so no”.





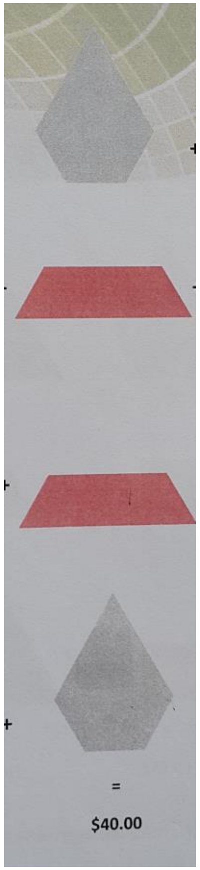
An example from Class B

Teacher: “guessing a random number didn’t work, but does anyone have a guess and check method that’s better?”

Student 1: “I knew that an even, plus an even, plus an even, plus an even is even. So the values for the second column have to be even”

Teacher repeats what the student says and writes it on the board

Teacher: “what should we do with this idea? Is this something that we’re certain about?”



An example from Class B

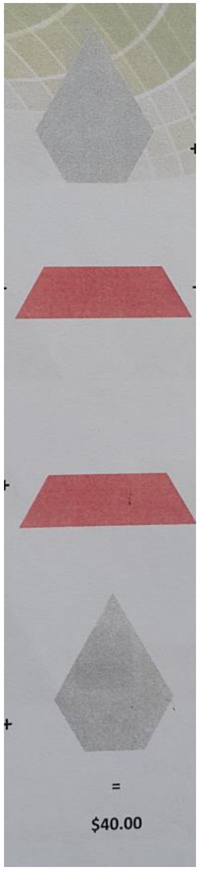
Mummering from the class, a few students call out "yes"

Teacher: "Sounds like a lot of yes's, but can we be sure? Who's going

Always, sometimes, never

Student 2: "Hang on! It's sometimes true but 1+1+1+1=4. That's four odds making an even, so you could do all odds…"

Student 1: "Oops. I guess I was just lucky that it worked"



Public evaluation and critique of mathematical thinking

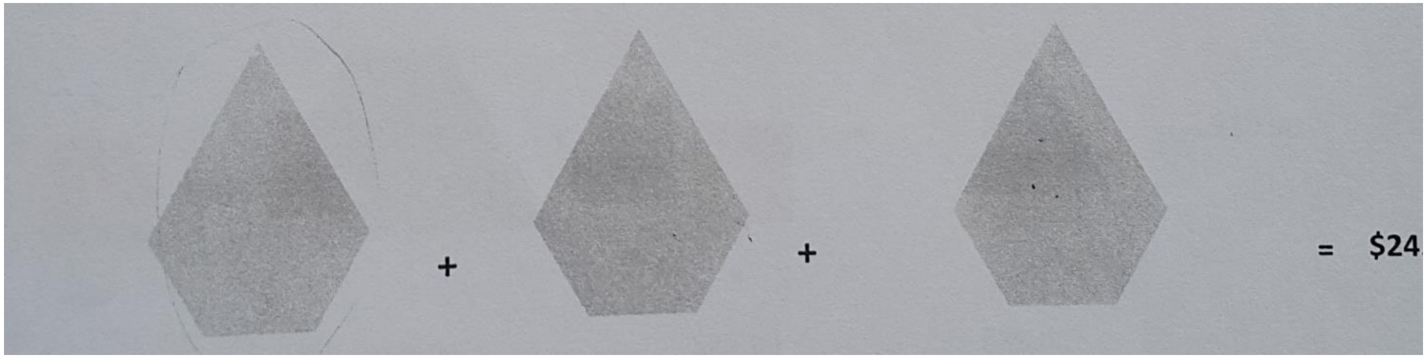
Class B held a public discussion to evaluate and critique ideas

- Ideas weren’t accepted until they’d been critiqued
- Students led this critique and had the opportunity to engage in mathematical reasoning
- Students used language and thought processes developed via always, sometimes, never activities
- All student ideas were critiqued

Student gives a correct response in Class A

Student 1: “we can divide 24 by 3 to work out that the white block is worth $8”

Teacher: “hmm. Good. That works. What next?”



Student gives a correct response in Class B

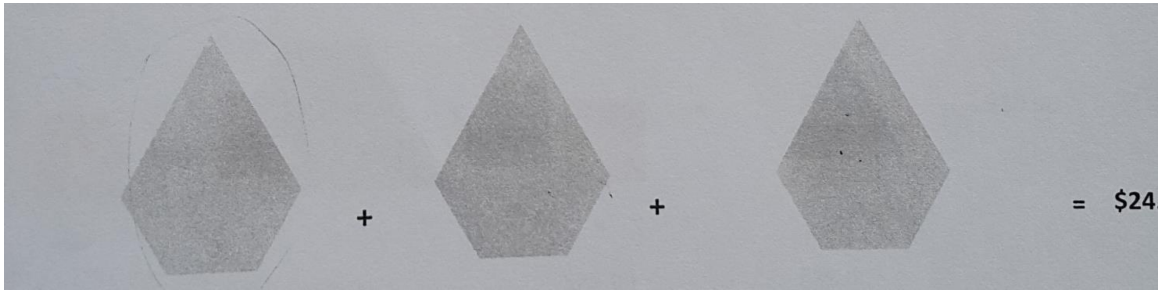
Student 1: “The white block must be 8 because three 8s are 24”

Teacher: “How does that work? Who can explain what she means?”

Student 2: “well, if you add something 3 times – it’s the same as doing times 3, so 3 times white is 24. That means 24 divided by 3 is 8”

Teacher: so you’re saying that we can divide by three for this one because we’re adding three things that are the same? Do we believe this?”

Class: “yes”



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Paraphrasing and polishing

The teacher in Class B tended to respond to students by paraphrasing what they just said:

- Student 2: “well, if you add something 3 times is the same as doing times 3, so 3 times white is 24. That means 24 divided by 3 is 8”
- Teacher: “so you’re saying that we can divide by three for this one because we’re adding three things that are the same?”

Often, student statements get a little bit of ‘polish’

- This helps facilitate the debate between students
- It buys time to think
- The teacher is still directing students towards ‘correct’ methods, but a bit of ‘polish’ guides them through the process of finding effective methods themselves

How much class time did each teacher get out of the task?

- Teacher A took about 15 minutes to get through the task
- Teacher B took slightly over 1 hour

By making evaluation a public process and building students critical communication skills, Teacher A is also reducing the amount of maths activities he needs to plan each week…

Getting critical communication happening in a class

An error that I used to make

I used to tell teachers that if they made evaluation a public process, then they could develop students' mathematical reasoning capacity

- The teacher of Class B said that it usually took a term to get students to start to reason in the way that was captured in the video

I get the answer quickly, then I'm done

I get the answer quickly, then I figure out how I'm going to prove my method works and I can explain it to others

We need to develop a language and structure that helps students know how to critique mathematical ideas or else the public evaluation of ideas can fall flat

Getting critical communication happening

1. Use an activity like *always, sometimes, never* to introduce ideas like counterexamples and start working through the logic of maths ideas
2. Set an assessment goal – e.g. write a mathematical reasoning comment for each of your students' reports
3. Try paraphrasing and polishing (but do step 1 too!)
4. Make evaluation of mathematical ideas a public process for 'correct' and 'incorrect' ideas

Looking for evidence of mathematical reasoning

- *If* ___ *then* ___ statements. Fundamental to formal logic
- Transfer of strategies between tasks
- Can build towards generalisations
- Can prove that an idea works

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But does it work with little children?

Research on reasoning in children

Inductive reasoning (proof by example) develops from an early age

The ability to process simple *if P then Q* type reasoning has been shown to develop between the ages of 5 and 7 in most children

So we shouldn't expect Foundation students' reasoning to mathematical reasoning to match 12 year olds' reasoning

(Hollister Sandberg & McCullough, 2010)

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How would younger children go with these?

Always sometimes never:

- An odd number plus an odd number is even
- 34 tens and 7 ones is the same as 3 hundreds and 47 tens
- If you count by 10s, the ones digit is always 0

Younger children will probably use examples more than older children, so they might use more pictures and blocks and then learn to explain how the pictures or blocks prove their point

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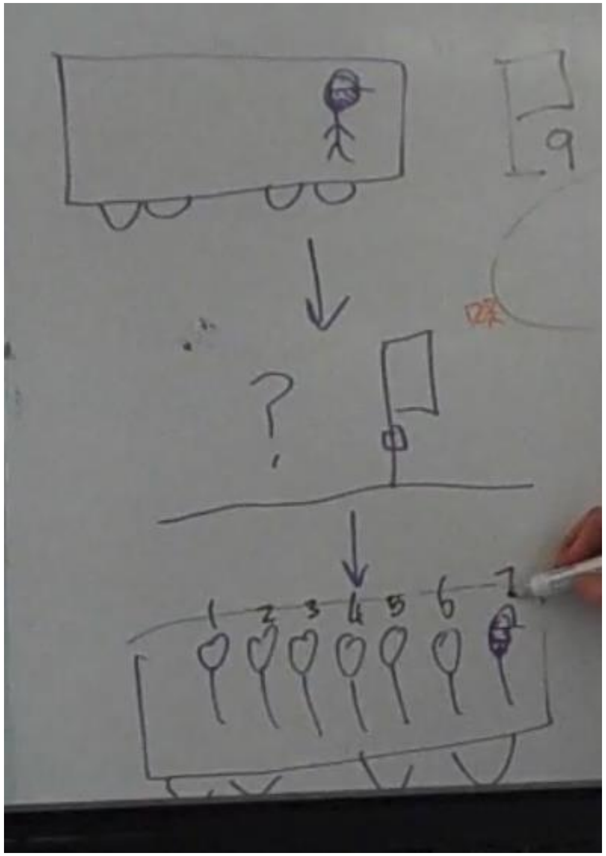
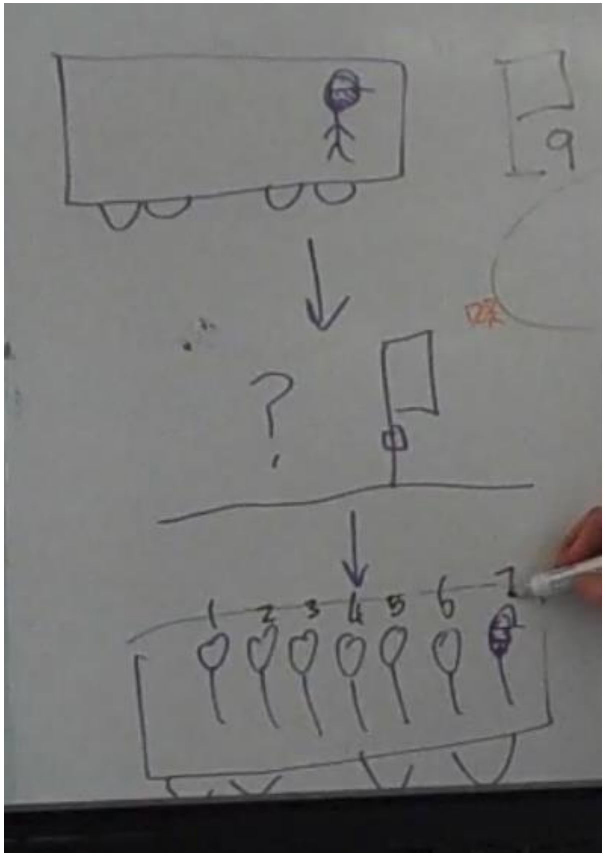
Public evaluation of student ideas

Here's an example from a Grade 1 class.

The problem:





A student answered that 7 people got on the tram. They were invited to join the teacher at the front.

The student spent 3 min explaining their reasoning

Student A: “you’ve made a mistake! It’s 6, but I can’t explain it but it’s wrong”.

Teacher: “if we really want to help, we’ll need to explain why it’s 6; how can we explain it?”

Student B: “we could draw it” (then tells the teacher what to draw)

Class spends 4 min proving why 6 is the correct answer

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Working question by question

With older children, the public evaluation stage can go for longer, so we can give students multiple questions to work through then evaluate

With younger children, working through one question, then evaluating, then a next question, then evaluating can stop the evaluate phase getting too long

In the tram problem, this meant that the teacher could give students an extension problem if they found the first 2 questions too easy and had concrete materials ready to help students that were still struggling after the first 2 questions

Why do we want to focus on mathematical reasoning?

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