Nell's and Norm's numbers

Derek Holton

dholton@unimelb.edu.au

Contact

- I am happy to be contacted by email (dholton@unimelb.edu.au)
- It may take me a day or two to answer email as I don't come to university every day.
- I am also happy to meet individuals or groups whenever or wherever it is mutually convenient.
- I'm happy to give anyone a copy of these slides

Aims

- My aims are to

  Provide an enjoyable experience

  Stimulate higher level thinking

  Provide a problem that almost all students can do

  Suggest some ideas for the classroom

Nell and Norm

Nell was playing with 2-digit numbers, like 73.

She reversed its digits to get 37.

Then she subtracted the smaller from the larger to get 36.

Then she did it again: reversed the digits and subtracted the smaller from the larger: 63 – 36 = 27.

For some reason she kept going. She told Norm what she had done. He tried with another number, but he was surprised to get the same result as Nell.

What had they found?

- By the way, for some reason if they got to a single digit answer, say, 6, they reversed it as 60 and them took 6 from 60.
- Try a few numbers and let me know what you think they found.

What did you get?

If there is in fact a pattern answer, what is it?

If there is no pattern, where were Nell and Norm going wrong?

Think again

- What happens **now** to 73 – 37; 63 – 36; and so on

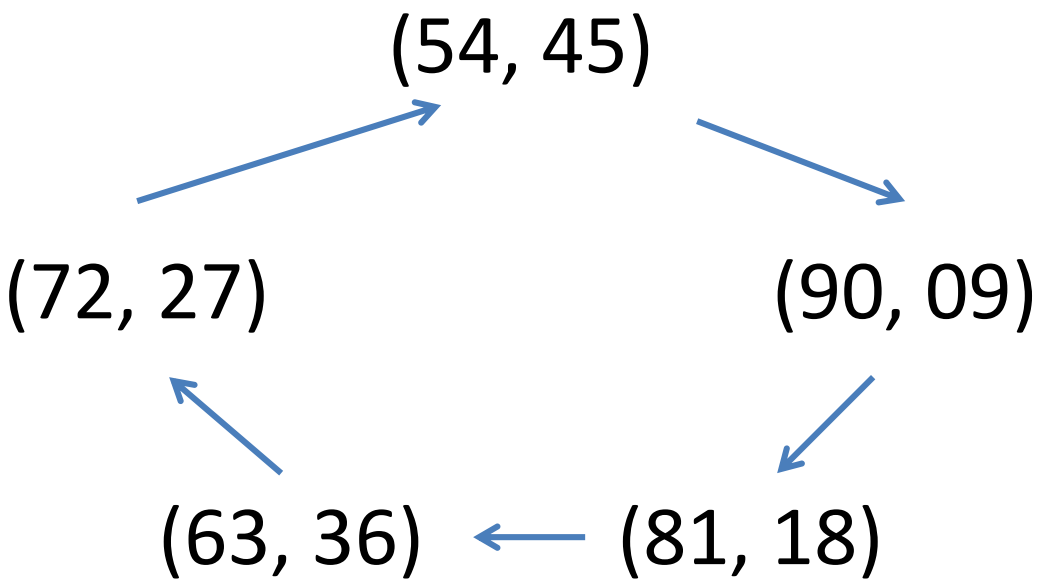
- Now try your favourite 2-digit number instead of 72 and repeat the reverse and subtract process again

Patterns and conjectures

What pattern(s) do you see?

What conjectures would you like to make?

A Cycle of Number Pairs



Justify

Does this cycle happen for all ‘seeds’ (starting numbers)?

Vote: Yes, no, I’ve no idea.

Conjecture.

Justify or find a counter-example.

The presence of 9

$10 - 01 = 1 \times 9$

$20 - 02 = 2 \times 9$

$$\begin{aligned} 73 - 37 &= (70 - 07) - (30 - 03) \\ &= 7 \times 9 - 3 \times 9 \\ &= 4 \times 9 \end{aligned}$$

So what?

So what?

- Change 7 and 3 to 6 and 1. The same ‘9’ thing happens only you get

$$61 - 16 = (60 - 06) - (10 - 01)$$
$$= 6 \times 9 - 1 \times 9$$
$$= 5 \times 9$$

One step more

We know that 37 and 73 go straight to the (63, 36) pair and 61 and 16 goes straight to (54, 45). Where do 53, 82, …, go?

OR

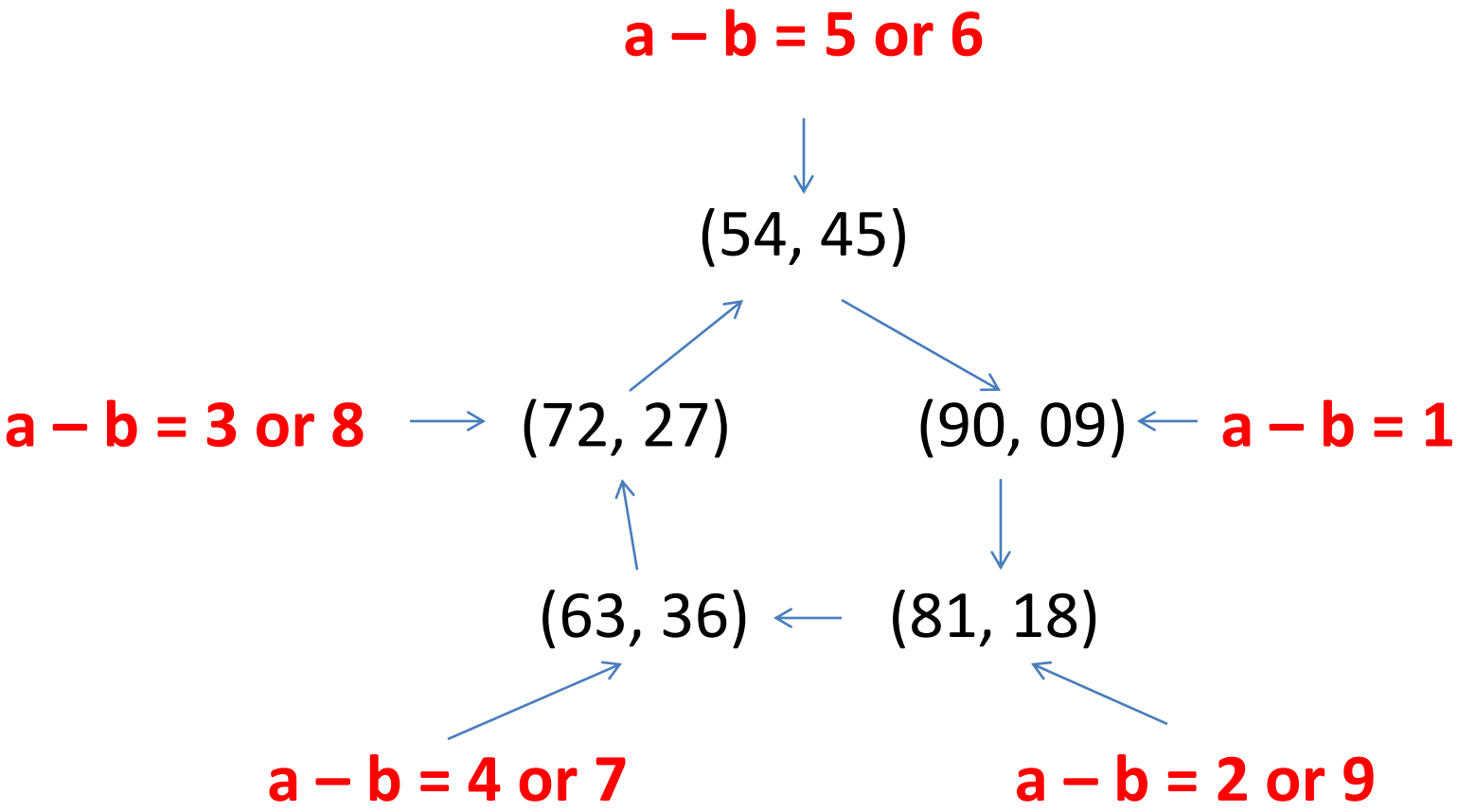
Can you say what numbers go to (63, 36) or (54, 45) or …?

What is the pattern? Where does the 2-digit number *ab* go?

The answer

- $ab - ba = 9(a - b)$
- So what numbers go to (54, 45)?
- And so on

The Extended Cycle



Extend further

What ideas do you have?

What shall we try?

What conjectures does that lead to?

3-digit numbers

- Do we get the same sort of thing for 3-digit numbers?

Class Use

Development much the same.

I use more algebra in the higher year levels.

I spend more time on group work.

I allow students to write results, conjectures and proofs on the board

Why use in class?

Genuine proficiency strand activity

- problem solving especially
- communication – working on board and in groups
- revision of 2-digit subtraction – fluency
- understanding through justification

Getting students to work together on something completely new

Entries and Exits

Exit 1: After first cycle of number pairs

Re-entry 1: After proof and heading to second cycle

Exit 2: After second cycle of number pairs

Re-entry 2: For extension to 3-digits

Exit 3: After proof of second cycle

Re-entry 3: After extension to 4-digits

Extend some more

- What do you think happens with 4-digit numbers?

  Same/not same/don't know

- Try it

Confession

- I know the answers for 4- and 5-digits BUT I had to use a computer/calculator!

- I DON'T KNOW THE ANSWERS FOR N-DIGITS WHERE N> 5.

- ***HELP!!***

On-line references

- http://nrich.maths.org/frontpage
- http://www.maths300.esa.edu.au/
- http://www.nzmaths.co.nz/

Book references

- Holton, D. & Lovitt, C. (2013). *Lighting Mathematical Fires 2*. Curriculum Press. Melbourne, Australia: ESA.
- Holton, D. (2012). *Problem Solving: The creative process*. Mathematical Association UK.
- Holton, D. (2013). *More Problem Solving: The creative process*. Mathematical Association UK.